It is evident from Eqs. (23), (29), (33), and (38) that the tangential stress at the plate increases with rise in τ and Gr and decreases with rise in σ. It is also seen that in the first, second, and fourth cases the flux breaks away after a certain time which depends on the Prandtl and Grashof numbers.

NOTATION

Gr, Grashof number $g\beta T_\omega \nu/u_0^3$; g, acceleration due to gravity; T, fluid temperature; T_0, plate temperature for $t < 0$; T_ω, change in plate temperature for $t = 0$; t, time; u, fluid velocity in the x direction; u_0, change in plate velocity for $t = 0$; u_1, dimensionless velocity (u/u_0); y, normal coordinate; α, thermal conductivity; β, thermal expansion coefficient; η, dimensionless coordinate (yu_0/ν); θ, dimensionless temperature $(T - T_0)/T_\omega$; ν, kinematic viscosity; σ, Prandtl number (ν/α); τ, dimensionless time $(u_0^2 t/\nu)$.

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CALCULATION OF THE HEATING OF POLYDISPERSE PARTICLES IN A GAS

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UDC 536.244

The problem of the heating of polydisperse particles in a gas is solved with allowance for the temperature field inside a particle and the variation of the gas temperature.

At the time $t = 0$ let an adiabatically closed volume of gas with a temperature $T(0)$ be uniformly filled with homogeneous, polydisperse, spherical particles having a temperature T_0. The problem consists in determining the average temperatures of the particles and the gas at any time. The energy equation is written in the form

$$c\rho \frac{dT}{dt} + 4\pi c_p \rho_p h_0 \int_0^\infty f(r_1)\left[\int_0^{r_1} r^2 \frac{\partial T_p}{\partial t}\, dr\right] dr_1 = 0. \quad (1)$$

The temperature of the particles is determined from the heat-conduction equation

$$\frac{\partial T_p}{\partial t} = a\nabla^2 T_p. \quad (2)$$

We choose the initial temperature of the particles as the origin of the temperature frame, and then the initial and boundary conditions take the form

$$T_p(t=0) = 0; \quad \left.\frac{\partial T_p}{\partial r}\right|_{r=0} = 0;$$

All-Union Scientific-Research Institute of Metallurgical Thermal Technology, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 38, No. 3, pp. 485-489, March, 1980. Original article submitted January 22, 1979.

0022-0841/80/3803-0285$07.50

$$\left.\frac{\partial T_p}{\partial r}\right|_{r=r_1} = \frac{\mathrm{Bi}}{r_1}\,[T - T_p(r_1,\ t)]. \tag{3}$$

With allowance for (3), the solution of (2) with T = const is known [1]:

$$\frac{T_p}{T} = 1 - \sum_{n=1}^{\infty} A_n \frac{r_1 \sin(\mu_n r/r_1)}{r\mu_n} \exp\left[-\mu_n^2 \frac{at}{r_1^2}\right] = \varphi(t), \tag{4}$$

$$A_n = \frac{2(\sin\mu_n - \mu_n \cos\mu_n)}{\mu_n - \sin\mu_n \cos\mu_n},$$

where μ_n are the roots of the equation $\tan\mu = -\mu/(\mathrm{Bi} - 1)$. We assume that the Biot number is independent of time. If T depends on time, then we find the solution with the help of Duhamel's theorem [1]

$$T_p = T(0)\,\varphi(t) + \int_0^t \frac{\partial T(t-t')}{\partial t}\,\varphi(t')\,dt'. \tag{5}$$

Using (4) and (5), the integral in (1) is converted to the form

$$\int_0^{r_1} r^2 \frac{\partial T_p}{\partial r}\,dr = \frac{r_1^3}{3}\left[T(0)\frac{\partial\varphi_1}{\partial t} + \int_0^t \frac{dT(t')}{dt'}\,\frac{\partial\varphi_1(t-t')}{\partial t}\,dt'\right],$$

where

$$\frac{\partial\varphi_1}{\partial t} = 6\frac{a}{r_1^3}\sum_{n=1}^{\infty} B_n \exp[-\mu_n^2 at/r_1^2],$$

$$B_n = \mathrm{Bi}^2/(\mu_n^2 + \mathrm{Bi}^2 - \mathrm{Bi}).$$

Using these expressions, from (1) we obtain

$$\frac{dT}{dt} + 6\varkappa a \int_0^{\infty} \frac{r_1^3}{r_0^3} f(r_1) \sum_{n=1}^{\infty} B_n \left[\frac{T(0)\exp[-\mu_n^2 at/r_1^2]}{r_1^2} + \frac{1}{r_1^2}\int_0^t \frac{dT(t')}{dt'} \exp[-\mu_n^2 a(t-t')/r_1^2]\,dt'\right] dr_1, \tag{6}$$

where $\varkappa = c'\rho'/c\rho$ is the ratio of the heat capacity of the particles to the heat capacity of the gas per unit volume of the gas suspension. This quantity depends weakly on temperature. We will neglect its temperature dependence. In (6) r_0 is the mass-average radius of the particles,

$$r_0^3 = \int_0^{\infty} r_1^3 f(r_1)\,dr_1. \tag{7}$$

From the condition of adiabaticity we write

$$c'\rho' \langle T_p \rangle + c\rho T(t) = (c'\rho' + c\rho)\,T_a, \tag{8}$$

where $\langle T_p \rangle$ is the average particle temperature at the time t.

We introduce the dimensionless variables $\tau = \beta^2 at/r_0^2$, $x = r_1/r_0$, where β is a scaling parameter introduced for convenience. We rewrite (6) in the form

$$\frac{dT}{d\tau} + \frac{6\varkappa}{\beta^2}\int_0^{\infty} x f(r_1) \sum_{n=1}^{\infty} B_n \left[T(0)\exp[-\mu_n^2\tau/\beta^2 x^2] + \int_0^{\tau} \frac{dT(\tau')}{d\tau'} \exp[-\mu_n^2(\tau-\tau')/\beta^2 x^2]\,d\tau'\right] dr_1 = 0. \tag{9}$$

We find the Laplace transform of this equation:

$$\frac{\bar{T}(s)}{T(0)} = \{s[1 + \varkappa y(s)]\}^{-1}, \tag{10}$$

where

$$y(s) = \frac{6}{\pi^2}\int_0^{\infty} x^3 f(r_1) \sum_{n=1}^{\infty} \frac{B_n}{z^2 + \mu_n^2/\pi^2}\,dr_1 \tag{11}$$

and $z = s\beta^2 x^2/\pi^2$. The smaller the Biot number, the faster the series in (11) converges; as $\mathrm{Bi} \to \infty$ the series is summed in closed form [2]:

$$\sum_{n=1}^{\infty} \frac{1}{z^2 + n^2} = \frac{1}{2z}\left[\pi\,\mathrm{cth}(\pi z) - \frac{1}{z}\right]. \tag{12}$$

Thus, the problem comes down to the calculation of the function y(s) and the inversion of the Laplace transformation. A convenient method of inverting the Laplace transformation was suggested by Salzer [3]:

TABLE 1. Table of Functions y(s)

s	β=10, Bi=∞	β=π, Bi=∞	β=1, Bi=∞	β=π, Bi=0,4	β=1, Bi=0,4
1	0,1897	0,4815	0,8316	0,1549	0,8094
2	0,1383	0,3752	0,7404	0,08452	0,5272
3	0,1144	0,3199	0,6781	0,05850	0,3973
4	0,09588	0,2843	0,6313	0,04484	0,3209
5	0,08981	0,2588	0,5943	0,03640	0,2701
6	0,08231	0,2393	0,5638	0,03066	0,2337
7	0,07644	0,2238	0,5380	—	0,2063
8	0,07167	0,2110	0,5159	—	0,1847

$$\frac{1}{2\pi i}\int_{\delta-i\infty}^{\delta+i\infty} e^{st}F(s)\,ds \simeq \sum_{k=1}^{m} A_k^m(t)\,F(k). \tag{13}$$

In this method one must know the values of the transform at the natural values $s = 1, 2, \ldots, m$. The functions $A_k^m(t)$ are tabulated in [3] for values of t from 0 to 10 with a step of 0.1-1 for m from 1 to 10. The larger m is the more precise the inversion. The convenience of the scaling parameter β consists in the fact that through its appropriate choice one can find the transform by Salzer's method for an arbitrary time and not only for those cases for which the table was compiled.

The calculations were made for a radial density distribution of the particles in the form

$$f(r_1) = Ar_1^{3/4}\, e^{-\gamma r_1} \tag{14}$$

for $r_0 = 105\ \mu m$. In this way a specific technical problem was solved, while the distribution (14) describes the experimental data with satisfactory accuracy. The function y(s) was calculated on a Mir-2 computer, while the inversion by Salzer's method was carried out on an Électronika S-50.

From (8) we find

$$\langle T_p \rangle / T_a = \frac{1+\varkappa}{\varkappa}(1 + T/T_0). \tag{15}$$

From this, using (10) for the Laplace transform, we obtain

$$\frac{\langle \bar{T}_p(s) \rangle}{T_a} = \frac{(1+\varkappa)\,y(s)}{s\,[1+\varkappa y(s)]}. \tag{16}$$

The results of the calculations are presented in Figs. 1 and 2. The case of $\varkappa = 0$ corresponds to heating of the particles at a constant gas temperature. We note that without the introduction of the scaling factor β^2 the inversion of the Laplace transformation by Salzer's method would have been impossible in a wide range of Fo. The method was used for $m = 6$ and, as a control on the accuracy, for $m = 8$ with a fixed β in some range of Fo. Further, by changing β^2 by an order of magnitude we made a calculation at the preceding end point and advanced farther along Fo. The values of the functions y(s) are presented in Table 1.

The results obtained can be used to calculate particle heating in a tuyere in a stream of hot gas. To calculate the Biot number it is sometimes necessary to allow for radiant heat exchange. The fine particles rapidly acquire the temperature of the gas and irradiate the large particles. In this case the effective coefficient of radiant heat transfer is $\lambda_r \approx 4\sigma(T + T_0)^3 r_1$. In addition, one must allow for the difference in the gas and particle velocities in determining the Nusselt number. All this leads to the fact that Bi, and hence μ_n, will depend on the particle radius, and the calculation of the function y(s) is somewhat complicated. In a specific engineering problem a calculation by this method gave a result diametrically opposite to that of a calculation using the two-temperature approximation. For example, limestone particles with a distribution (14) are hardly heated in a time of $5 \cdot 10^{-3}$ sec when $\varkappa = 0$. A calculation under the assumption that the coefficient of thermal conductivity of the particle material is infinitely high leads to almost complete heating in this case for particles with a radius of 10^{-4} m.

The stated problem can be solved by the numerical methods developed in [4, 5] in a more general statement with allowance for the temperature dependence of the thermophysical properties and for the time dependence of the Biot number. Although the proposed method has less generality, it requires less computational work, and in some cases it provides the possibility of obtaining a closed analytical result. For example, with a constant gas temperature and Bi → ∞ we obtain

$$\frac{\langle T_p \rangle}{T} = 1 - \frac{6}{\pi^2}\sum_{n=1}^{\infty}\int_0^{\infty}\frac{f(r_1)}{n^2}\exp\left[-\pi^2 a n^2 t/r_1^2\right]dr_1. \tag{17}$$

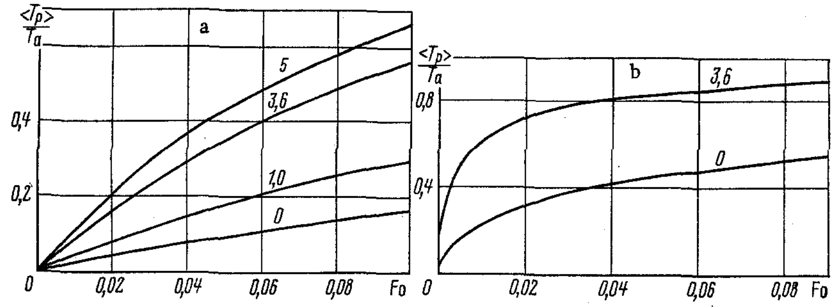

Fig. 1. Average particle temperature as a function of dimensionless heating time at different values of the parameter $\varkappa$: a) Bi = 0.4; b) Bi = ∞.

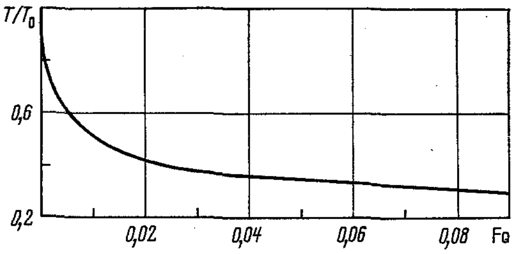

Fig. 2. Dependence of dimensionless gas temperature T/T(0) on Fourier number at $\varkappa$ = 3.6 and Bi = ∞.

Let the radial distribution of the particles have the form

$$f(r_1) = Ar_1^2 \exp[-r_1^2/r_m^2], \tag{18}$$

where r_m is the radius corresponding to the maximum of f; $A = 4/r_m^3\sqrt{\pi}$. Performing the integration and summation in (17), with allowance for (18) we obtain

$$\frac{\langle T_p \rangle}{T} = \frac{12}{\pi^2}\int_0^{\tau_1}\frac{d\tau}{e^{2\sqrt{\tau}}-1}, \tag{19}$$

where

$$\tau_1 = \pi^2 at/r_m^2. \tag{20}$$

If $\tau_1 \ll 1$, then from (19) we have

$$\langle T_p \rangle / T = \frac{12}{\pi^2}\sqrt{\tau_1}. \tag{21}$$

When $\tau_1 \gg 1$,

$$\frac{\langle T_p \rangle}{T} = 1 - \frac{6}{\pi^2}(1 + 2\sqrt{\tau_1})\, e^{-2\sqrt{\tau_1}}. \tag{22}$$

NOTATION

c, c_p, specific heats at constant pressure for gas and particle material, respectively; ρ, ρ_p, densities of gas and particle material; $c'\rho'$, heat capacity of particles per unit volume of gas suspension; T, $T_p(r)$, gas temperature and particle temperature reckoned from initial particle temperature T_0; $\varkappa = c'\rho'/c\rho$, ratio of heat capacity of particles to heat capacity of gas; $f(r_1)$, distribution density of particles by radii r_1, normalized to one; $a = \lambda_p/c_p\rho_p$, coefficient of thermal diffusivity; Bi, Biot number; $Fo = at/r_0^2$, Fourier number; r_0, mass-average radius of particles; s, parameter of Laplace transformation; < >, symbol for average value; β, scaling parameter; T_a, temperature of gas and particles as $t \to \infty$; t, time; n_0, number of particles per unit volume.

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KINETICS OF THE NEUTRALIZATION OF STATIC ELECTRICITY IN APPARATUS CONTAINING TWO-PHASE SYSTEMS OF GAS AND SOLID PARTICLES

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UDC 541.182:537.226

The mechanisms of neutralization of static electricity of disperse systems when ions of the opposite sign are present in the gaseous medium are discussed.

In the design of neutralizers of static electricity for industrial apparatus containing two-phase systems of gas and solid particles the calculation of the performance of these neutralizers acquires great importance. The value of this parameter can be determined by analyzing the process of neutralization of the charges of particles of the product being treated in the presence of ions of the opposite sign in the gas–air medium of the working volume of the apparatus. Let us consider a charged spherical particle over which a stream of ions of the opposite sign flows (Fig. 1).

Under these conditions the variation of the charge of a particle is described by the equation

$$dq/dt = e \int_S \bar{j}_n d\bar{S}. \quad (1)$$

The flux density is defined as [1]

$$j_n = -nkE_\Sigma + D \operatorname{grad} n. \quad (2)$$

The neutralization of the charges of a particle takes place in accordance with Eq. (2) as a single process, but to simplify the solution we will consider two mechanisms separately: a) ion motion directed toward the surface of the particle due to the electric field; b) ion motion due to diffusion.

Let us consider the first mechanism. The following forces act on an ion which is near a charged particle:

$$\bar{E}_\Sigma = \bar{E}_e + \bar{E}_{pol} + \bar{E}_c + \bar{E}_m + \bar{E}_{equ}. \quad (3)$$

Let us find the components of these voltages on the vector $\overline{dS}$:

$$1)\ E_e = |\bar{E}_e| \cdot \cos\theta = \frac{qNr_i}{2\varepsilon_0} \cos\theta. \quad (4)$$

To simplify the calculations we assume that $\bar{E}_e$ is the same at different points

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 38, No. 3, pp. 490-497, March, 1980. Original article submitted January 2, 1979.